

EXPERIMENTAL DETERMINATION OF NONLINEAR

ROTARY STABILITY DERIVATIVES

by

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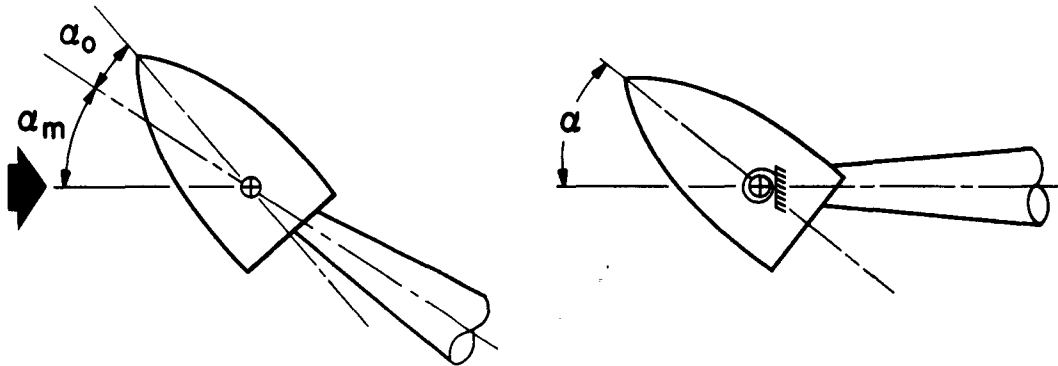
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## INTRODUCTION

Nonlinearities now appear regularly in all phases of aerodynamic research. Experimenters in dynamic stability have particular reason to be concerned about them, since the methods available for extracting stability derivatives from dynamical data for the most part do not acknowledge the presence of nonlinearities. When the aerodynamic characteristics are nonlinear, these methods continue to give results, but it becomes very difficult to say precisely what the results mean. Consider, for example, the two widely used experimental methods sketched below.



In the first method, the sting is brought to a fixed mean angle of attack  $\alpha_m$  and the model is forced to oscillate harmonically about  $\alpha_m$  with a small amplitude  $\alpha_o$ . The work done per cycle is measured and identified with the aerodynamic damping derivative  $C_{m\dot{\alpha}} + C_{m\dot{\alpha}_o}$ . In the second method, the model is displaced from zero angle of attack against the resistance of a spring and then released. The decay of the free oscillation about zero is measured and identified with the damping derivative  $C_{m\dot{\alpha}} + C_{m\dot{\alpha}_o}$ . When the aerodynamic moment is a linear function of its variables, the same value for the damping derivative is obtained from both experiments. When the moment is nonlinear, two entirely different results can be obtained. Neither result is necessarily wrong; each merely reflects the particular method in use, which in turn reflects a particular facet of the underlying nonlinear phenomenon.

Consider the analyst who must use these results to analyze the motion of a vehicle in flight. If the motion he anticipates consists of small deviations from a trim condition, the first result may be applicable. If the motion he anticipates consists of large-amplitude oscillations around zero, the second result may be

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applicable. But, suppose he anticipates the second and has results only for the first. Or, suppose he can anticipate neither motion. He has no assurance at all that the results at hand have any meaning in his problem. Obviously, this situation imposes a severe limitation on the usefulness of wind-tunnel measurements of stability coefficients.

It is clear that if this limitation is to be overcome, results from different facilities and different experimental techniques must be made to correlate, so that they may be used interchangeably. In other words, the experimental results must be freed from dependence on the particular method used to determine them. A recent report (ref. 1) addressed to this problem is briefly summarized in the remainder of this note.

#### GENERAL FORM FOR THE AERODYNAMIC PITCHING MOMENT

Before the results of an experiment can be freed from the method used to determine them, the true form of the equation governing the motion must be established. This form will, of course, depend on the form of the aerodynamic forces and moments. The problem is, therefore, to write the aerodynamic forces and moments in a form which is sufficiently general to apply to any of a wide variety of possible motions. In the linear case, one achieves this by writing the forces and moments as a sum of terms involving stability derivatives. The question then arises, do stability derivatives even exist in the nonlinear case? Based upon the analysis of reference 1 the answer is yes, with certain strict reservations. For the longitudinal case, involving arbitrary variations of angle of attack  $\alpha$  and angle of pitch  $\theta$ , the aerodynamic pitching moment has the following form

$$C_m(t) = C_m(\infty; \alpha(t)) + \dot{\theta}(t) \frac{l}{V_0} C_{m_q}(\infty; \alpha(t)) + \dot{\alpha}(t) \frac{l}{V_0} C_{m_{\dot{\alpha}}}(\alpha(t)) \quad (1)$$

The terms have the following meaning: The first term  $C_m(\infty; \alpha(t))$  is the familiar steady-state pitching-moment coefficient due to angle of attack, evaluated at each instant as though the instantaneous value of  $\alpha$ ,  $\alpha(t)$ , were fixed for an infinite time at that value. The infinity symbol as used in this notation is merely a reminder that the flow is fixed for all time, that is, is steady at the particular value of  $\alpha$  under consideration. The second term  $C_{m_q}(\infty; \alpha(t))$  is the rate of change with  $\dot{\theta}$  of the pitching-moment coefficient that would be measured in a steady flow,

evaluated at  $\dot{\theta} = 0$  with  $\alpha$  fixed for an infinite time (thus the infinity sign again) at the instantaneous value  $\alpha(t)$ . The third term  $C_{m\dot{\alpha}}$  is defined by means of an integral: It is the area bounded by the indicial pitching-moment curve and the final value of the curve, the indicial response being evaluated with  $\alpha$  fixed at the instantaneous value  $\alpha(t)$  and with  $\dot{\theta}$  fixed at zero.

It must be emphasized that the definitions just given for the terms in equation (1) are precise and unique; they arise from a rigorous analysis correct to the first order in frequency. Within this order of approximation, alternative forms or definitions would not be rigorously justifiable. Thus, for example, note that each of the terms in equation (1) shows a dependence only on the instantaneous angle of attack  $\alpha(t)$ ; to show a dependence on  $\dot{\alpha}(t)$ ,  $\dot{\theta}(t)$ , or higher derivatives of  $\alpha$  and  $\theta$  would not be justifiable. Equation (1) is the desired general form which should underlie all special cases and apply to all particular motions.

#### FORM OF THE AERODYNAMIC PITCHING MOMENT IN A SPECIAL CASE

Further specifications of the form of  $C_m(t)$  are possible in particular cases. Let us consider one of these for a wind-tunnel experiment. In most wind-tunnel experiments, the model is pinned to a fixed point at its axis of rotation. Then  $\dot{\theta} = \dot{\alpha}$ . For the wind-tunnel experiment, therefore,  $C_{m\dot{\alpha}}$  and  $C_{m\dot{\theta}}$  simply add, and  $C_m(t)$  takes the form

$$C_m(t) = C_m(\infty; \alpha(t)) + \dot{\alpha}(t) \frac{l}{V_0} [C_{m\dot{\alpha}}(\infty; \alpha(t)) + C_{m\dot{\theta}}(\alpha(t))] \quad (2)$$

Now suppose that the model under study is known from static tests to have a static pitching-moment curve which is representable as an odd cubic in  $\alpha$  over a substantial range of  $\alpha$ . Then  $C_m(\infty; \alpha(t))$  has the form of equation (3) below, and the rate of change of  $C_m$  with  $\alpha$ ,  $C_{m\alpha}(\infty; \alpha(t))$ , has the form of an even quadratic in  $\alpha$  (eq. (4)).

$$C_m(\infty; \alpha(t)) = \alpha(a + b\alpha^2) \quad (3)$$

$$C_{m\alpha}(\infty; \alpha(t)) = a + 3b\alpha^2 \quad (4)$$

Since the boundary conditions which yield  $C_{m\dot{\alpha}}$  are very similar to those which yield  $C_{m\alpha}$ , it is very reasonable to assume that  $C_{m\dot{\alpha}}$  likewise will show no more than an even quadratic dependence on  $\alpha$ .

Under similar conditions, it can be argued that  $C_{m\dot{\alpha}}$  also will be an even quadratic in  $\alpha$ . Then, so is the sum  $C_{m_q} + C_{m\dot{\alpha}}$ . Thus,

$$C_{m_q}(\infty; \alpha(t)) = c + d\alpha^2 \quad (5a)$$

$$C_{m\dot{\alpha}}(\alpha(t)) = e + f\alpha^2 \quad (5b)$$

and

$$C_{m_q} + C_{m\dot{\alpha}} = h_0 + h_2\alpha^2 \quad (5c)$$

Alternatively,  $C_{m_q} + C_{m\dot{\alpha}}$  can be written as a linear function of the steady-state parameter  $C_{m_\alpha}(\infty; \alpha(t))$  as shown below.

$$C_{m_q} + C_{m\dot{\alpha}} = C + DC_{m_\alpha}(\infty; \alpha(t)) \quad (6)$$

This particularly simple form for the damping coefficient has certain consequences which could be checked experimentally.

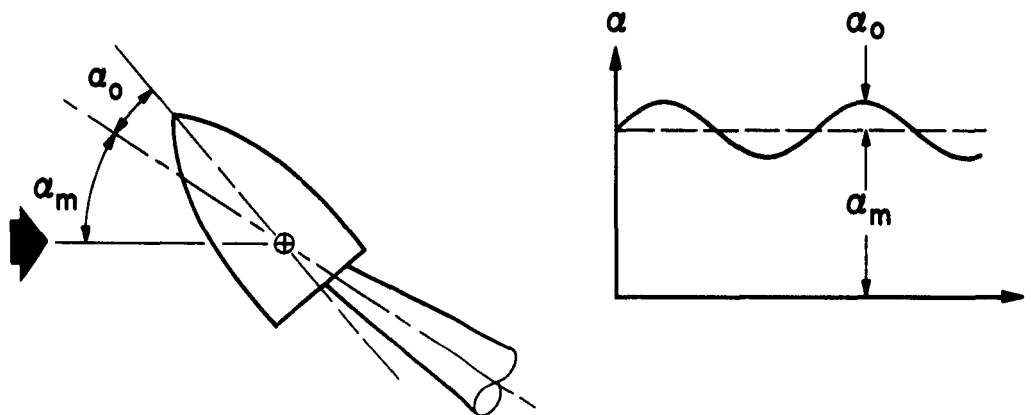
#### EXPERIMENTAL CORRELATIONS

Now let us consider the main objective, correlation of experimental results. We wish to show that, at least in certain cases, knowledge of the form of the pitching-moment coefficient enables one to free the results for damping coefficient obtained by a particular experimental method from dependence on that method. If the results from two very different experimental methods were both freed from dependence on their respective methods, then those results should correlate. Results which correlate can of course be used interchangeably. If results from two very different motions can be used interchangeably, then we are assured that either result is a general one; that is, one which is applicable to any motion whatever.

Let us assume that the model under consideration fulfills the requirement discussed in the last section, namely, that its static pitching-moment curve be a cubic in  $\alpha$ . Then we anticipate that the damping coefficient should be a quadratic in  $\alpha$  or, alternatively, a linear function of the static pitching-moment curve slope  $C_{m_\alpha}(\infty; \alpha(t))$ . Suppose this is true. Then this is the form we expect whenever a particular experimental result for damping coefficient is freed from the method of obtaining it. We next consider how this might be achieved in specific experiments.

# EXPERIMENTAL TECHNIQUES

Consider the two very different experimental methods illustrated in the previous sketch. The first, forced oscillations about a fixed mean angle of attack  $\alpha_m$ , is shown again below.



The work required to drive the model is measured and equated to the work done by the aerodynamic damping moment. In the linear case, the work measured can be identified with a single, unique value of the aerodynamic damping coefficient  $C_{m_q} + C_{m_{\dot{\alpha}}}$ . In the nonlinear case, this is no longer true. Here, when the work measured is identified with the work done by aerodynamic damping, the result obtained depends on both the mean angle  $\alpha_m$  and the oscillation amplitude  $\alpha_o$ ; that is, the result depends on the method used to obtain it. The objective then is to free the result from this dependence.

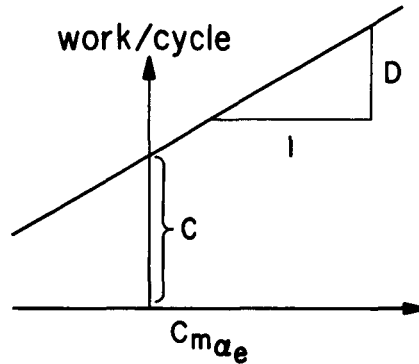
When the aerodynamic damping coefficient depends linearly on the instantaneous value of the static pitching-moment curve slope (i.e., eq. (6)), the work done by damping will have the form shown in equation (7) where  $C$  and  $D$  are the same  $C$  and  $D$  in equation (6).

$$\text{Work/Cycle} = C + DC_{m_{\alpha e}} \quad (7)$$

where

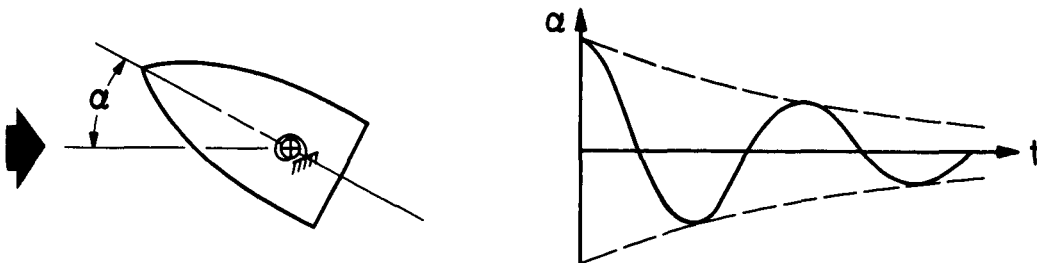
$$C_{m_{\alpha e}} = \frac{\partial C_m}{\partial \alpha} (\infty; \alpha_m) + \frac{\alpha_o^2}{8} \frac{\partial^3 C_m}{\partial \alpha^3} (\infty; \alpha_m) \quad (8)$$

That is, the work done by damping becomes a linear function of an effective value of  $C_{m\alpha}$ ,  $C_{m\alpha_e}$ , where  $C_{m\alpha_e}$  can be evaluated from static data and equation (8). Hence, plotting the work done by damping against  $C_{m\alpha_e}$  gives a straight line.



Note that the same straight line will be obtained for any value of oscillation amplitude  $\alpha_0$ . The slope and zero intercept of the line give the values of  $C$  and  $D$ . When these are substituted in equation (6), together with the true static curve  $C_{m\alpha}(\infty; \alpha(t))$ , the form obtained for the damping coefficient  $C_{m\dot{\alpha}} + C_{m\alpha}$  should then be general - applicable to any motion whatever. This expectation can be checked by an independent experiment.

The second method, to be used as a check on the generality of the preceding result, is the well-known free-oscillation technique, shown again below.



The equation of motion for this system is

$$\frac{I\ddot{\alpha}}{q_0 S l} = C_m(t) + \text{spring terms} \quad (9)$$

With the same model as the one used in the first experiment, the static pitching-moment curve is a cubic in  $\alpha$  and the damping coefficient is a quadratic in  $\alpha$ , so that the pitching moment (eq. (2)) takes the form

$$C_m(t) = \alpha(a + b\alpha^2) + \frac{\dot{\alpha}l}{V_0} (h_0 + h_2\alpha^2) \quad (10)$$

Combining equations (9) and (10) yields the following for the equation of motion

$$\frac{I\ddot{\alpha}}{q_0 S l} = \alpha(a + b\alpha^2) + \frac{\dot{\alpha}l}{V_0} (h_0 + h_2\alpha^2) + \text{spring terms} \quad (11)$$

In this form, the equation of motion is a combination of the Duffing equation and the Van der Pol equation. An approximate solution to the equation is obtainable from the Kryloff-Bogoliuboff method. Fitting the solution to the measured motion history enables the evaluation of the four constants  $a, b, h_0, h_2$ . These have a known correspondence with the constants  $C$  and  $D$  used in the first experiment; that is,  $C$  and  $D$  can be expressed as

$$\left. \begin{aligned} C &= h_0 - \frac{h_2 a}{3b} \\ D &= \frac{h_2}{3b} \end{aligned} \right\} \quad (12)$$

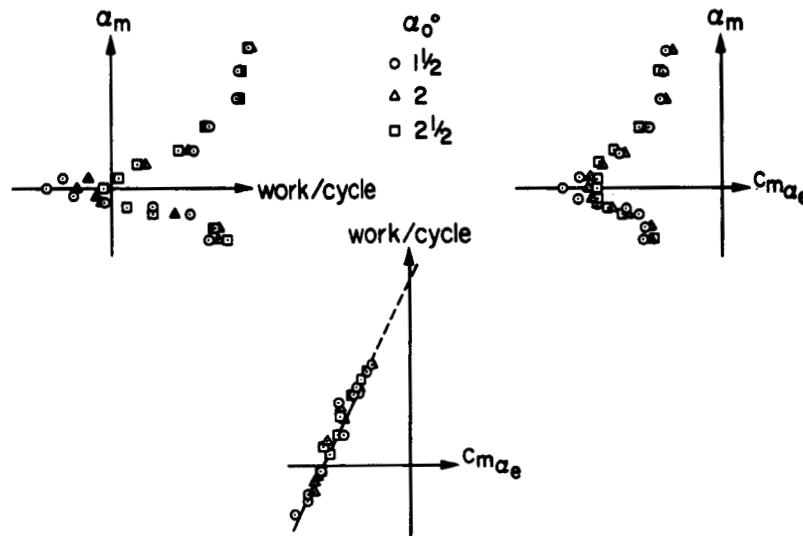
If the values of  $C$  and  $D$  computed from equations (12) agree with those obtained in the first experiment, then we have shown that two very different experimental methods can be made to yield results which correlate. This in turn assures that the form which correlates is a general one, applicable to any motion whatever.

#### SUGGESTED EXPERIMENT

The above discussion suggests an obvious experiment: Select a model whose static pitching moment  $C_m(\infty; \alpha)$  can be represented as a cubic over a substantial range of  $\alpha$ . Carry out the two experiments in the manner just described. If the results for  $C_{m_q} + C_{m_\alpha}$  can be



expressed as a quadratic in  $\alpha$ , and if the values of C and D agree between the two experiments, this will count both as a confirmation of the theory and a useful means of widening the applicability of the experimental results. Finally, we point out that a linear relationship between  $C_{m_q} + C_{m_{\alpha}}$  and  $C_{m_{\alpha}}$  can be expected to obtain under a number of circumstances other than those discussed here (i.e.,  $C_{m_{\alpha}}$  a quadratic function of  $\alpha$ ). For example, a linear relationship between  $C_{m_q} + C_{m_{\alpha}}$  and  $C_{m_{\alpha}}$  can be expected to obtain whatever the variation of  $C_{m_{\alpha}}$ , provided that  $C_{m_q}$  is either small or essentially independent of  $\alpha$ . The sketch below shows an experimental result which confirms this statement for a model whose value of  $C_{m_q}$  is known to be small. Note that  $C_{m_{\alpha e}}$  is a complicated function of  $\alpha$ ; nevertheless, the relationship between work done and  $C_{m_{\alpha e}}$  is quite obviously linear.



#### REFERENCE

1. Tobak, Murray, and Pearson, Walter E.: A Study of Nonlinear Dynamic Longitudinal Stability. NASA TR R-209, 1964.